Collaborative Distance Discovery Learning

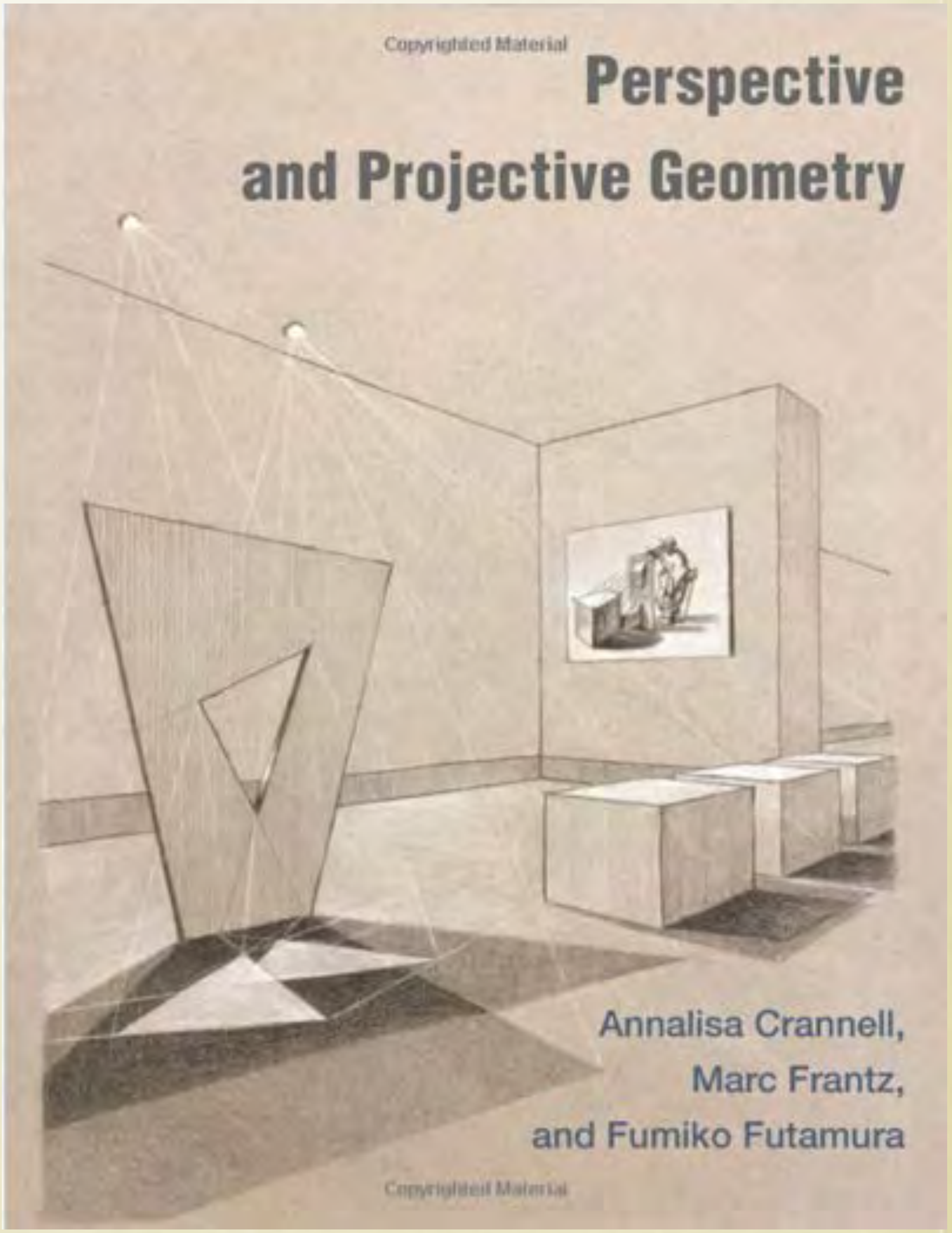
Food for Discussion

Gordon Williams

June 16, 2020

Context

- MATH 305: Geometry
  - Upper division math course exploring connections between classical and modern geometries. Emphasis on axiomatics and mathematical proof.
  - Small class organized around the text *Perspective and Projective Geometry*.
  - Before COVID-19 closure, class time was about 75% small group work.
  - After closure, I needed to figure out how to translate this discovery based learning course online.



Discovery Based Learning

- *Not* "I, We, You" but "You, Y'all, We".
- Emphasizes *methodology*.
- Focus is on problems as a *path* to learning content, vs. problems as a path to *solidifying* content.
- This was my first time teaching this way whole hog.
- Course emphasized connections between geometry and perspective in art and vision.

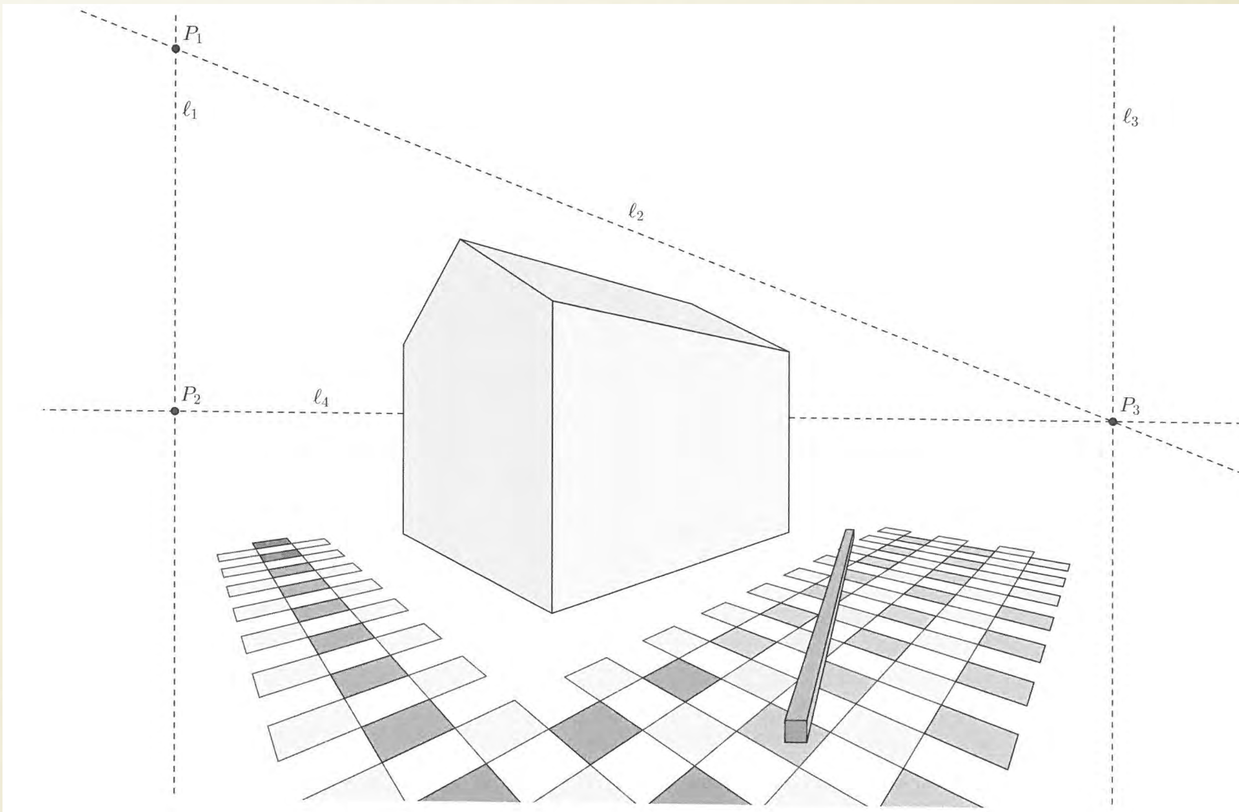


FIGURE 5.0: Extending the space we live in. [For use with the EXTENDED EUCLIDEAN SPACE: TO INFINITY AND BEYOND module.]

The Book

Note that even though we call $P_{[\![\ell]\!]}$ a "point," it is not actually a point in $\mathbb{R}^2$. The definition above is different from the one defining, for example, a "vanishing point" of ℓ, where we declared which of the points already in existence in $\mathbb{R}^3$ lies in the plane ω on a line of sight parallel to ℓ. In contrast, an *ideal point* brings something into $\mathbb{E}^2$ that does not exist in $\mathbb{R}^2$. Sometimes mathematicians call this a "formal definition," because it *forms* a new object.

The next set of definitions tells us how ideal points "connect" to points and lines in $\mathbb{R}^2$.

Definition The *extended plane*, $\mathbb{E}^2$, consists of the points of the Euclidean plane $\mathbb{R}^2$ together with the collection of ideal points of lines in $\mathbb{R}^2$ such that the following conditions hold.

- Elements of $\mathbb{E}^2$ are *points*; a point in $\mathbb{E}^2$ is either an *ordinary* point $P \in \mathbb{R}^2$ or an *ideal* point $P_{[\![\ell]\!]}$ for some line $\ell \subset \mathbb{R}^2$.
- A line in $\mathbb{E}^2$ is either the *ideal line* ℓ_∞ (which we define to be the union of all ideal points in $\mathbb{E}^2$) or an *ordinary line* $\ell = \ell_0 \cup P_{[\![\ell_0]\!]}$ (obtained from the union of the points of a Euclidean line ℓ_0 together with the ideal point $P_{[\![\ell_0]\!]}$ of that line.)

1. Think of the ground plane in Figure 5.0 as $\mathbb{E}^2$. That is, ignore lines in the house for now, and pay attention only to lines on the ground. Which sets of lines in the ground plane of Figure 5.0 contain the ideal points named below:
   - the ideal point whose perspective image is P_2?
   - the ideal point whose perspective image is P_3?
2. In Figure 5.0, somebody left a long bar lying on the ground. Where is the image of the ideal point for the long edges of this bar?

What can we conclude about the space $\mathbb{E}^2$? Let's explore. When you see a statement listed as [T/F], you should decide whether the statement is true or false, and then provide a convincing argument that you are right. You should support your arguments using the definitions above and the already-proved statements from previous chapters.

3. [T/F] There exists a pair of distinct lines $k, \ell \in \mathbb{E}^2$ with $k \cdot \ell = \emptyset$.

   ["$k \cdot \ell$" is the intersection of lines k and l.]

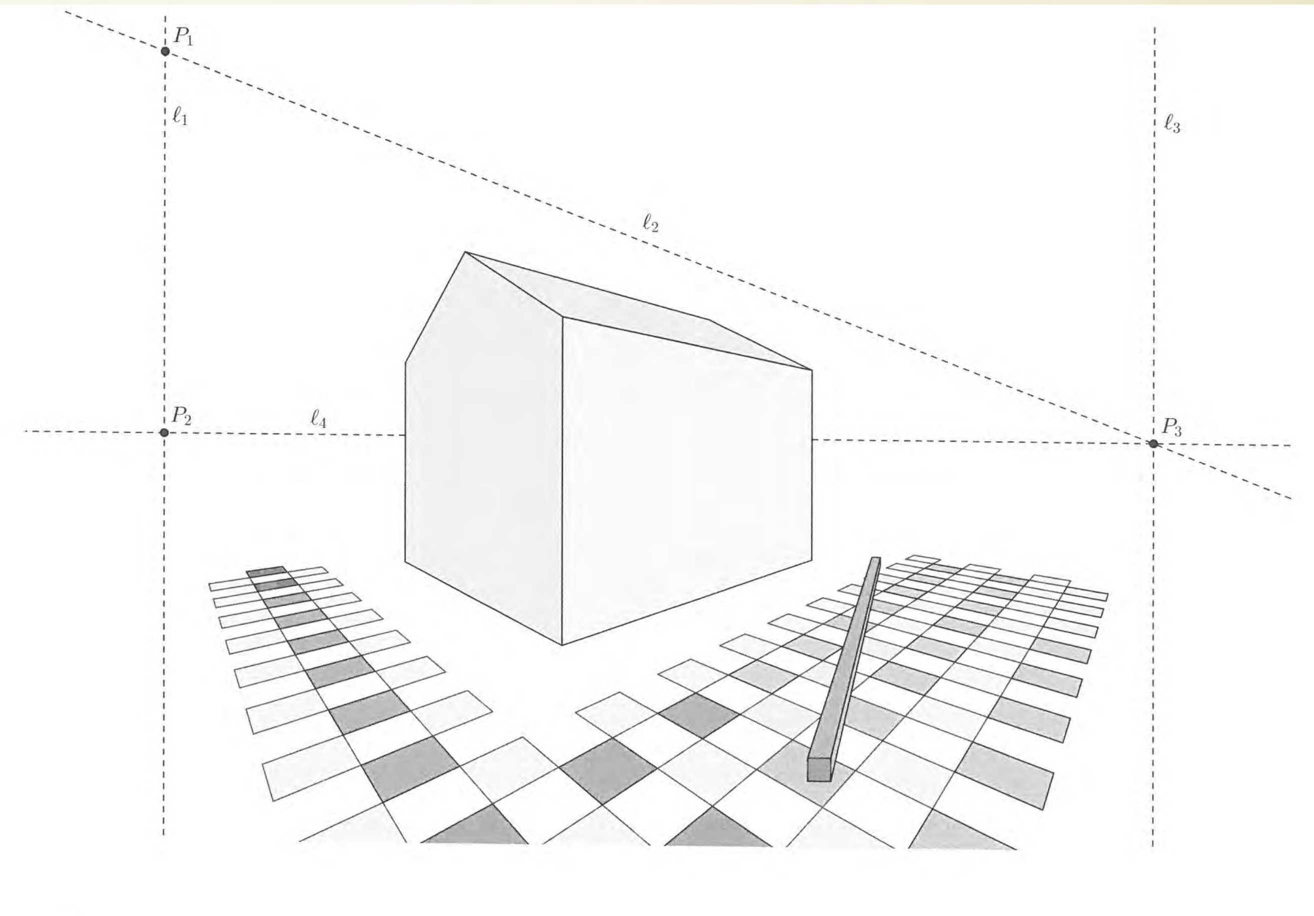


FIGURE 5.0: Extending the space we live in. [For use with the EXTENDED EUCLIDEAN SPACE: TO INFINITY AND BEYOND module.]

4. [T/F] There exists a pair of distinct lines $k, \ell \in \mathbb{E}^2$ such that $k \cdot \ell$ contains a single point.

5. [T/F] It is possible for a pair of distinct lines in $\mathbb{E}^2$ to have two or more points of intersection.

6. [T/F] Every pair of distinct lines in $\mathbb{E}^2$ intersects in one and only one point.

7. [T/F] If P is an ordinary point and Q is an ideal point, then PQ exists.

["PQ" is the line containing points P and Q.]

5. We know that the image of a line in $\mathbb{R}^3$ could be another line. The physical world does not always correspond to the abstract mathematical setting, however. Draw top and side views and/or 3-D sketches for a geometrical setting, or give physical examples from a real-world setting, to show whether the image ℓ' could take the following forms:

(a) $\emptyset$;

(b) a point;

(c) a line segment;

(d) a ray;

(e) a line with one point missing;

(f) a line with two points missing; or

(g) an ellipse.

Transition Objectives

- Continuity of experience
- Sense of connection
- Maintain or reduce workload on students

How I Moved Online

- Zoom: Breakout groups
- Gradescope: "Online Assignment" for worksheets
  - You can try a sample by enrolling at www.gradescope.com using the course entry code 93NP8J.
- Gradescope: All assessments submitted online.

What It Looks Like

Q1 7.1.1
2 Points

Q1.1
1 Point

What is the shadow of the lamppost?

- (●) A point
- ( ) A line segment
- ( ) A line
- ( ) A plane

Q1.2
1 Point

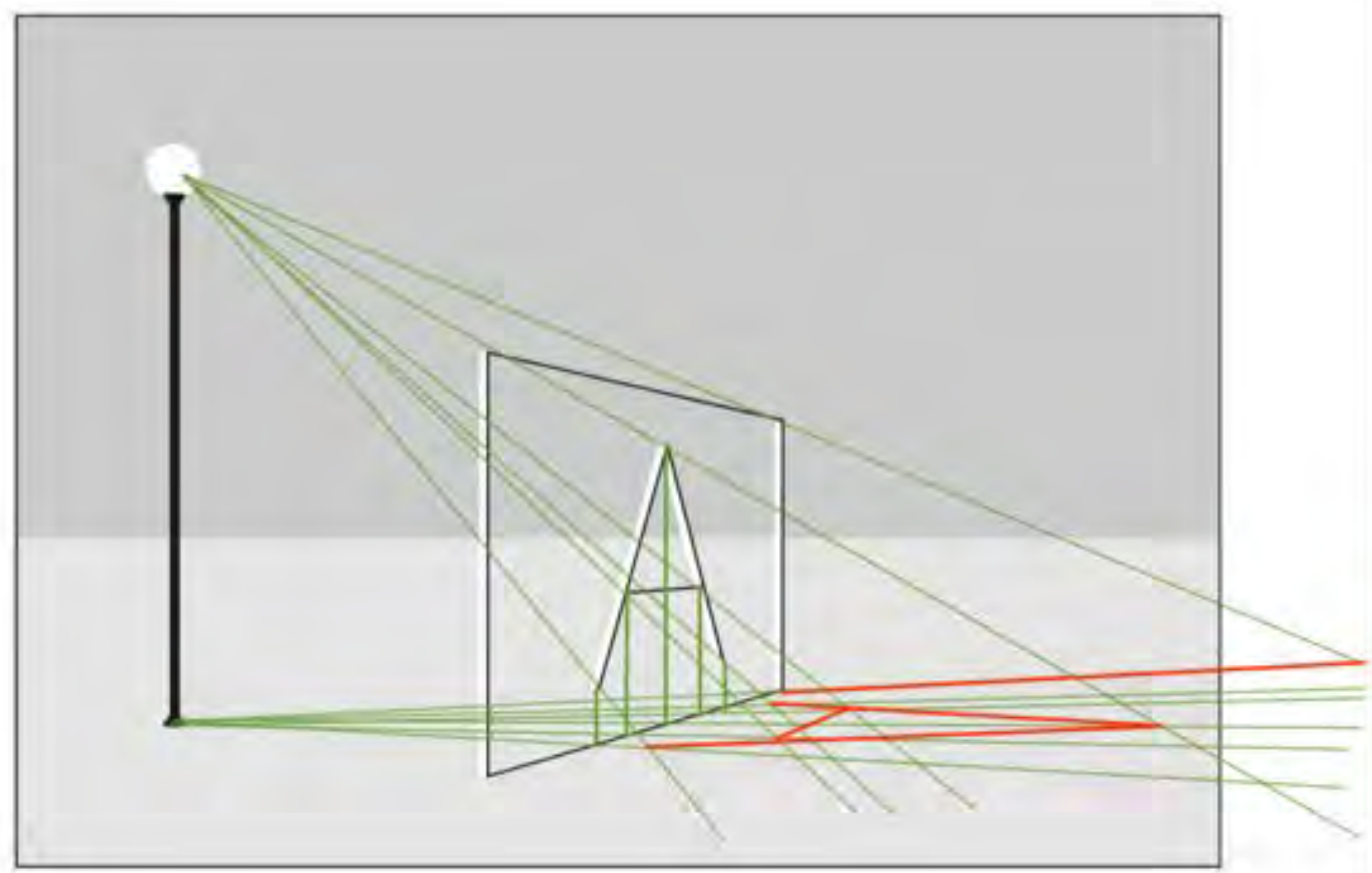
Where is the shadow?

Directly underneath the lamp post.

Q4 7.1.4-5
1 Point

Upload your drawing of where the shadow of the letter "A" and the back vertical edge of the glass goes.

▾ Lamppost.pdf　　Download



Q5 7.1.6
1 Point

Are the triangle part of the "A" and its shadow perspective from a point?

yes

If so, what point?

The light

Label that point O in your picture.

Q6 7.1.7
1 Point

Again, are the triangle and its shadow perspective from a line?

Yes

If so, which line?

The bottom of the pane of glass.

Label that line o in your picture.

Q7 7.1.8
1 Point

Consider the meshes $\mathcal{A}$ and $\mathcal{A}'$ and $\mathcal{E}$ and $\mathcal{E}'$. Are either of these pares of meshes perspective from a point? From a line?

- [ ] $\mathcal{A}$ and $\mathcal{A}'$ are perspective from a point.
- [x] $\mathcal{A}$ and $\mathcal{A}'$ are perspective from a line.
- [x] $\mathcal{E}$ and $\mathcal{E}'$ are perspective from a point.
- [ ] $\mathcal{E}$ and $\mathcal{E}'$ are perspective from a line.

Q3.2
1 Point

Why?

TRUE. The corresponding line segments will always be parallel since no rotation has occurred and the translation moves along some direction which is the same for every point so the lines joining vertices will always be parallel to that direction.

Q4 8.3.4-5
1 Point

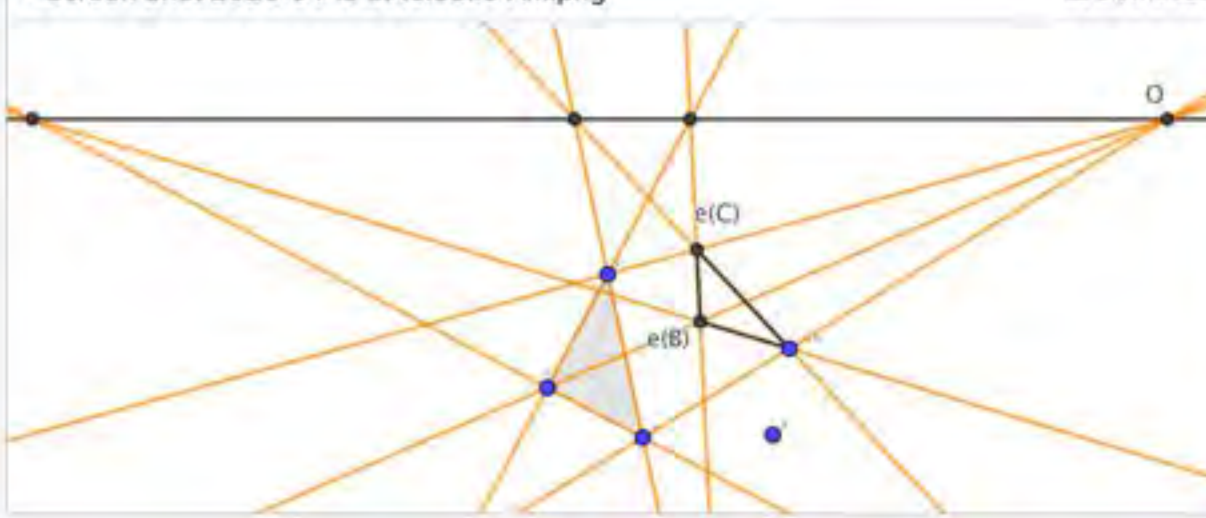
Locate the vanishing point of the line $Ae(A)$. Label that point O.
Extend the line segment AC to a line. Which point on the line AC is fixed by the elation?

It would be the vanishing point for the line AC.

Q5 8.6-8
1 Point

Sketch the image $e(AC)$.
Locate the point $e(C)$.
By repeating the ideas from the last few steps, sketch the image of the translated triangle $e(ABC) = e(A)e(B)e(C)$. Check your solution by making sure the triangles are perspective from both the point O and the line of the horizon.

▾ Screen Shot 2020-04-13 at 10.55.10 AM.png   Download



Q17 8.1.19

1 Point

Read the book...

Given a point Y coplanar with $$X$ and o, describe a method for determining the location of Y'.

Build lines $\overline{OY}$ and $\overline{YX}$. Consider $P_o = \overline{YX} \cdot o$. Now $Y' = \overline{P_o X'} \cdot \overline{OY}$.

Q18 8.1.20

1 Point

Given a line k coplanar with $$X$ and o, determine the location of k'.

Consider distinct points P and Q where $P \in k$ and $Q = k \cdot o$ (by definition $Q = Q'$). Now build the line $\overline{PX}$ and $\overline{PO}$. Note $P_o = \overline{PX} \cdot o$. Note, $P' = \overline{PO} \cdot \overline{X' P_o}$. Thus $k' = \overline{P'Q}$.

Practices Before

- (YOU, Y'ALL) I monitored group discussions, gave hints, and corrected confusions. Sometimes as a class, sometimes just with groups.
- (YOU, Y'ALL) Students were expected to take notes in their texts capturing their discoveries.
- (WE) Students were asked to present and discuss selected problems at the board. I gave feedback on presentation style.
- (WE) Summary sheet.

Practices After

- (YOU, Y'ALL) Students worked in groups in Zoom, I moved back and forth monitoring discussion and checking answers.
- (YOU, Y'ALL) Students were now much more careful about writing answers collaboratively, and would debate wording. Complete sentences used.
- (WE) Summary sheet.
- (WE) End of class discussion mostly gone. Instead, I graded every problem students worked.

Observations

- Students were less likely to gloss over the details when working on the online versions of the worksheets.
- Summary sheet was less important (and less successful) since students had the shared notes of their group's more careful answers.
- Progress was usually *faster* but *felt slower* to students.
- Students missed being able to get ideas from overhearing conversations.

Observations

- Students caught more of their own mistakes. Discussions richer.
- Group submissions encouraged better collaboration and made grading quick. Grading scheme was granular but simple. Feedback was detailed (but rarely needed).
- In both of my classes I maintained synchronous classes. Student feedback for this choice was almost uniformly positive, often effusively so.

Conclusions

- Print worksheets constrain students' responses. Online are more flexible.
- I'm planning to use Gradescope Online Assignments for group activities in more classes in future, not just synchronous distance delivery.
- Designing good discovery learning activities is hard, error prone work. If building my own from scratch I'd want an in-person class the first time.